Correlation by Simulation: An estimation and inferential tool for correlation amidst estimable measurement error.

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Abstract

A general purpose tool is presented for calculating correlation estimates and corresponding inferential intervals in the context of estimable measurement error. The new method, dubbed Correlation by Simulation ($CorSim$), is founded on a simple simulation method that

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lends a combination of flexibility of application and inferential capacity heretofore unknown in traditional correction procedures. Monte Carlo simulation reveals that $CorSim$ outperforms traditional approaches to correlation estimation and inference.

1 Introduction

Few empirical measurements can be made without some degree of measurement error. When estimating the correlation between two variables, x and y, across a sample of subjects from a population, it is well known that if the measurement of either variable contains error, this error tends to lead to raw correlation values that underestimate the true correlation, ρ, between x and y across the population. However, repeated measurement of x and y within each subject facilitates characterization of the distribution of error associated with the measurement of each variable within each subject, information that can then be used to improve the rigor of correlational analysis. In this spirit, Spearman (1904) suggested that a corrected estimate of ρ may be obtained by

$$Est.(\rho) = r_{xy} \times \gamma_{rel} \tag{1}$$

where r_{xy} is the observed correlation between x and y, and γ_{rel} is a correction factor defined as

$$\gamma_{rel} = \frac{1}{\sqrt{r_{xx} \times r_{yy}}} \tag{2}$$

where r_{xx} and r_{yy} are the observed reliabilities of x and y, respectively. The values of r_{xx} and r_{yy} are typically derived from reliability estimation procedures such as the split-half method. It is unclear, however, whether traditional inferential statistics remain applicable to the product of this method, here dubbed the γ_{rel} method. Further, the γ_{rel} method bears the inconvenient property that it may occasionally (and in some cases, often; see present results) yield estimates of ρ that fall outside the acceptable range of correlation statistics (-1 to +1).

An alternative approach, developed by Liu et al. (1978) and here dubbed the γ_{var} method, suggests that observations of both within and between-subjects variance can be used to estimate ρ:

$$Est.(\rho) = r_{xy} \times \gamma_{var} \tag{3}$$

where the correction factor, γ_{var}, is defined as

$$\gamma_{var} = \sqrt{(1 + \frac{\lambda_x}{\bar{k}_x})(1 + \frac{\lambda_y}{\bar{k}_y})} \tag{4}$$

where $\bar{k}_x$ and $\bar{k}_y$ represent the mean number of observations of x and y, respectively, per subject and λ_x is defined as

$$\lambda_x = \frac{\bar{\omega}_x}{\beta_x} \tag{5}$$

where β_x is the between-subjects variance observed in x and $\bar{\omega}_x$ is the mean within-subject variance observed in x, with a similar definition of λ_y. In contrast to the γ_{rel} method, inferential procedures have been developed (Rosner and Willett, 1988) for interpreting correlation estimates provided by γ_{var} method. However, as with the γ_{rel} method, the γ_{var} method is also susceptible to the problem of producing estimates outside the acceptable range.

The present work attempts to provide researchers with a new tool for estimation and inference in correlation research, founded on a simple simulation procedure and therefore dubbed Correlation by Simulation ($CorSim$). This report outlines the application of the $CorSim$ procedure to a simple experimental design and validates this application via Monte Carlo simulation.

2 Correlation by Simulation

Consider a data set involving a repeated measurement of x and y within each of N subjects. From this data one may calculate:

1. $\bar{x}$, a N item vector containing the mean value of x measured within

each subject.

2. $\bar{y}$, a N item vector containing the mean value of x measured within each subject.
3. r_{xy}, a single value describing the raw correlation between x and y, calculated from $\bar{x}$ and $\bar{y}$.
4. β_x, a single value describing the variance of values in $\bar{x}$.
5. β_y, a single value describing the variance of values in $\bar{y}$.
6. k_x, a N item vector representing the number of observations of x made within each subject.
7. k_y, a N item vector representing the number of observations of y made within each subject.
8. ω_x, a N item vector representing the variance of x measured within each subject.
9. ω_y, a N item vector representing the variance of y measured within each subject.

Items 4 through 9 form what is here dubbed the “variance model”.

2.1 Estimating ρ

Obtaining these quantities we can ask: Given the observed variance model and raw correlation, what is the most likely value of ρ? Turning this question around we can ask the more tractable question: Given the observed variance model, what value of ρ makes the observed raw correlation maximally likely? The $CorSim$ method proposes to answer this question by exploring the distribution of raw correlation values obtained through simulations that generate simulated data sets derived from the combination of the observed variance model and a series of candidate values for ρ. This procedure can be more specifically outlined as follows:

1. A candidate value for ρ is chosen.
2. A covariance matrix, σ, defined as

   $$\sigma = \begin{pmatrix} \beta_x & \rho\sqrt{\beta_x \beta_y} \\ \rho\sqrt{\beta_x \beta_y} & \beta_y \\ & \end{pmatrix}$$

   is used to define a bivariate normal distribution
3. N samples are drawn from the bivariate normal distribution to define τ_x and τ_y, each a N item vector representing simulated true values of x and y, respectively, for each simulated subject.
4. For each simulated subject i, τ_{x_i} is used to define the center of normal distribution with a variance defined by ω_{x_i}, from which k_{x_i} random simulated measures of x are sampled and collapsed to a mean.
5. For each simulated subject i, τ_{y_i} is used to define the center of normal

distribution with a variance defined by ω_{y_i}, from which k_{y_i} random simulated measures of y are sampled and collapsed to a mean.

6. The correlation between these mean simulated values for x and y is then calculated and stored.

7. Steps (a) through (e) are repeated a large number of times, yielding a distribution of simulated raw correlation values

8. Given this distribution, the likelihood of the observed raw correlation is estimated.

Using these steps, we may search for the value of ρ that yields highest likelihood of obtaining the observed raw correlation. The resultant value of ρ serves as our estimate of the true ρ describing the relationship between x and y.

2.2 Confidence intervals on the estimate of ρ

Point estimates of statistical parameters are rarely of particular excitement unto themselves; research is typically more interested in the range of values over which the true parameter is likely to fall. The $CorSim$ method provides a straightforward means of estimating such a range, and in the example that follows 95% confidence intervals will be generated. The procedure is nearly identical to that outlined above with the exception that step 8 is replaced by the calculation of the percentile that the observed correlation achieves within the distribution of simulated raw correlation values. We then search for two values of ρ; the value of ρ that produces a distribution of simulated raw correlation values within which the observed correlation falls at the 2.5%ile, and the value of ρ that produces a distribution of simulated raw correlation values within which the observed correlation falls at the 97.5%ile.

3 Monte Carlo Validation

3.1 Simulation Design

To validate the accuracy of the *CorSim* method, a Monte Carlo simulation was programmed in R (R Development Core Team, 2007) and performed on high performance computing facilities provided by the Shared Hierarchical Academic Research Computing Network (www.sharcnet.ca).[1] Within a given Monte Carlo iteration, N paired samples were drawn from a multivariate normal distribution with a covariance matrix defined by:

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \\ & \end{pmatrix}$$

Each resulting data point was used as the center of a distribution with a

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[1] While validation was performed using high performance computing facilities, application of the *CorSim* method to a single data set on on a contemporary personal computer typically yields results within a matter of minutes.

variance of λ from which k values were drawn. Thus, on each iteration N artificial subjects were created, and for each k observations were made in each of 2 conditions.

The simulation explored the estimates of ρ generated by 4 procedures: the $CorSim$ method, raw correlation (r), the γ_{rel} method, and the γ_{var} method. The simulation also explored the confidence intervals generated by 3 methods: the $CorSim$ method, Fisher's r-to-z transformation as applied to the raw correlation and γ_{rel} corrected estimates, and Rosner and Willett's (1988) formulas for calculating confidence intervals for γ_{var} corrected estimates.

Reliabilities provided to the γ_{rel} method were estimated using an odd-even split-half correlation that was then corrected using the Spearman-Brown prediction formula. Within the $CorSim$ method, the likelihood of r_{xy} given the observed variance model and a given candidate value of ρ was estimated by rounding each simulated correlation value (produced in step 7 of the description above) to two digits and counting the proportion of simulated correlation values that were equal to the two-digit-rounded value of r_{xy}. Search across candidate values of ρ was achieved across 3 steps:

1. Test values in the interval of -1 to 1 in increments of 0.05 then define $Est.(\rho)_1$ as the candidate value of ρ that yields the greatest likelihood of r_{xy}.

2. Test values in the interval of $Est.(\rho)_1 - .2$ to $Est.(\rho)_1 + .2$ in increments of 0.005 then define $Est.(\rho)_2$ as the candidate value of ρ that yields the greatest likelihood of r_{xy}.

3. Test values in the interval of $Est.(\rho)_2 - .01$ to $Est.(\rho)_2 + .01$ in increments of 0.0005 then define $Est.(\rho)_3$ as the candidate value of ρ that yields the greatest likelihood of r_{xy}.

$Est.(\rho)_3$ is then used as the $CorSim$ estimate of ρ.

Four factors thought to influence estimation were completely crossed: true correlation ($\rho = \{0, .2, .5, .8, 1\}$), number of subjects ($N = \{10, 30, 100\}$), number of observations per subject ($k = \{10, 30, 100\}$), and ratio of within-subject variance to between-subjects variance ($\lambda = \{.1, 1, 10\}$). For each combination of these factors, 2000 Monte Carlo experiments were performed. Stability of results was assessed by repeating this process and visually comparing the two sets of results; finding no substantial differences, both sets were combined into a single set for presentation in this paper.

3.2 Simulation Results

The first performance measure assessed was the “rejection rate”, which is the rate at which each estimation method yielded estimates of ρ that exceeded the bounds of -1 to +1. Tables 1-3 show the rejection rate for the γ_{rel} and γ_{var} methods (no other method yields rejected estimates). From these tables

Table 1: Rejection Rate (%) at $\lambda = .1$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	γ_{rel}	0	0	0	0	15
		γ_{var}	0	0	0	0	18
	30	γ_{rel}	0	0	0	0	3
		γ_{var}	0	0	0	0	3
	100	γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
30	10	γ_{rel}	0	0	0	0	2
		γ_{var}	0	0	0	0	2
	30	γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
	100	γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
100	10	γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
	30	γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
	100	γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0

it can be seen that rejection rates are approximately equivalent between the γ_{rel} and γ_{var} methods, with the exception of the $\lambda = 10$ data set, where the γ_{rel} method yields substantially higher rejection rates than theγ_{var} method. Across methods, rejection rate appears to be positively proportional to ρ and λ, and negatively proportional to N and k. For subsequent analyses, all values that fall outside the bounds of -1 to +1 were removed from the data set.

Tables 4-6 show estimation bias (multiplied by 100 for visual convenience) of all four methods under investigation. In these tables, we see the expected at-

Table 2: Rejection Rate (%) at $\lambda = 1$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	γ_{rel}	0	0	1	7	49
		γ_{var}	0	0	1	5	50
	30	γ_{rel}	0	0	0	1	37
		γ_{var}	0	0	0	1	43
	100	γ_{rel}	0	0	0	0	16
		γ_{var}	0	0	0	0	19
30	10	γ_{rel}	0	0	0	0	41
		γ_{var}	0	0	0	0	32
	30	γ_{rel}	0	0	0	0	24
		γ_{var}	0	0	0	0	22
	100	γ_{rel}	0	0	0	0	3
		γ_{var}	0	0	0	0	2
100	10	γ_{rel}	0	0	0	0	31
		γ_{var}	0	0	0	0	13
	30	γ_{rel}	0	0	0	0	8
		γ_{var}	0	0	0	0	3
	100	γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0

Table 3: Rejection Rate (%) at $\lambda = 10$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	γ_{rel}	46	48	54	67	76
		γ_{var}	6	8	10	20	28
	30	γ_{rel}	11	11	17	34	60
		γ_{var}	1	2	5	16	43
	100	γ_{rel}	0	1	1	7	48
		γ_{var}	0	0	0	5	51
30	10	γ_{rel}	11	13	21	40	61
		γ_{var}	0	0	0	3	9
	30	γ_{rel}	0	0	1	11	50
		γ_{var}	0	0	0	1	23
	100	γ_{rel}	0	0	0	0	41
		γ_{var}	0	0	0	0	32
100	10	γ_{rel}	0	0	2	19	51
		γ_{var}	0	0	0	0	0
	30	γ_{rel}	0	0	0	1	46
		γ_{var}	0	0	0	0	6
	100	γ_{rel}	0	0	0	0	30
		γ_{var}	0	0	0	0	13

tenuation of the observed raw correlation as a function of λ, which is notably unaffected by increasing N, a recourse commonly assumed to improve estimation accuracy. On the other hand, it is apparent that the performance of all methods does improve with increased values of k. All estimation methods appear to show greater negative bias with increasing values of λ and with increasing values of ρ. All correction methods appear to outperform the raw correlation estimate. With the lowest value of N, the $CorSim$ method slightly outperforms γ_{rel} and γ_{var} methods, particularly at high values of ρ and λ. On the other hand, at the higher values of N the γ_{rel} method appears to outperform the $CorSim$ and γ_{var} methods, again particularly at high values of ρ and λ.

An ideal estimator of the 95% confidence interval should yield intervals in which ρ falls 95% of the time; intervals that capture ρ at a rate less than 95% represent intervals that are too narrow, yielding overly liberal inferential tests, while intervals that capture ρ at a rate greater than 95% represent intervals that are too broad, yielding overly conservative inferential tests. The performance of an estimate of the 95% confidence interval can thereby be characterized by investigating the ρ capture bias, that is the degree to which the observed ρ capture rate differs from the expected rate of 95%. ρ capture bias for all four methods is depicted in Tables 7-9, revealing that the $CorSim$ method performs quite well and consistently across the full parameter space. Fisher's r-to-z transformation as applied to the raw correlation estimates per-

Table 4: Bias ($[Est.(\rho) - \rho] \times 100$) at $\lambda = .1$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	$CorSim$	0	1	1	-1	0
		r	0	-1	-2	-2	-1
		γ_{rel}	0	-1	-2	-2	0
		γ_{var}	0	-1	-2	-2	0
	30	$CorSim$	0	0	1	-1	0
		r	0	-2	-2	-2	0
		γ_{rel}	0	-2	-2	-2	0
		γ_{var}	0	-2	-2	-2	0
	100	$CorSim$	0	1	2	0	0
		r	0	-1	-1	-2	0
		γ_{rel}	0	-1	-1	-2	0
		γ_{var}	0	-1	-1	-2	0
30	10	$CorSim$	0	1	1	0	0
		r	0	0	-1	-1	-1
		γ_{rel}	0	0	-1	0	0
		γ_{var}	0	0	-1	0	0
	30	$CorSim$	1	1	1	0	0
		r	1	-1	-1	-1	0
		γ_{rel}	1	-1	-1	-1	0
		γ_{var}	1	-1	-1	-1	0
	100	$CorSim$	-1	2	1	0	0
		r	0	0	-1	-1	0
		γ_{rel}	0	0	-1	-1	0
		γ_{var}	0	0	-1	-1	0
100	10	$CorSim$	0	1	0	0	0
		r	0	-1	-1	-1	-1
		γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
	30	$CorSim$	0	1	0	0	0
		r	0	0	0	0	0
		γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
	100	$CorSim$	0	1	1	0	0
		r	0	0	0	0	0
		γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0

Table 5: Bias ($[Est.(\rho) - \rho] \times 100$) at $\lambda = 1$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	$CorSim$	1	0	-1	-2	-3
		r	0	-3	-6	-9	-10
		γ_{rel}	1	-1	-2	-2	-5
		γ_{var}	1	-1	-2	-2	-5
	30	$CorSim$	0	0	0	-2	-1
		r	0	-2	-4	-5	-4
		γ_{rel}	0	-2	-2	-2	-2
		γ_{var}	0	-2	-2	-2	-1
	100	$CorSim$	0	1	0	-1	0
		r	0	-1	-4	-2	-1
		γ_{rel}	0	-1	-3	-2	0
		γ_{var}	0	-1	-3	-1	0
30	10	$CorSim$	0	1	0	-2	-2
		r	0	-2	-5	-8	-9
		γ_{rel}	0	0	0	-1	-2
		γ_{var}	0	0	-1	-1	-2
	30	$CorSim$	0	1	1	-1	0
		r	0	-1	-2	-3	-3
		γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
	100	$CorSim$	0	2	1	0	0
		r	0	0	-1	-1	-1
		γ_{rel}	0	0	-1	-1	0
		γ_{var}	0	0	-1	-1	0
100	10	$CorSim$	0	0	-1	-1	-1
		r	0	-2	-5	-8	-9
		γ_{rel}	0	-1	0	0	-1
		γ_{var}	0	-1	-1	-1	-1
	30	$CorSim$	0	1	0	0	0
		r	0	-1	-2	-3	-3
		γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0
	100	$CorSim$	0	1	1	0	0
		r	0	0	0	-1	-1
		γ_{rel}	0	0	0	0	0
		γ_{var}	0	0	0	0	0

Table 6: Bias ($[Est.(\rho) - \rho] \times 100$) at $\lambda = 10$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	$CorSim$	1	-7	-15	-25	-33
		r	1	-11	-26	-42	-52
		γ_{rel}	1	-7	-20	-32	-41
		γ_{var}	0	-9	-19	-33	-41
	30	$CorSim$	-1	-2	-5	-9	-12
		r	0	-6	-15	-22	-27
		γ_{rel}	-1	-3	-7	-11	-16
		γ_{var}	-1	-3	-8	-12	-17
	100	$CorSim$	1	1	0	-2	-3
		r	1	-2	-6	-9	-10
		γ_{rel}	1	0	-2	-3	-5
		γ_{var}	1	0	-2	-2	-5
30	10	$CorSim$	-1	-5	-13	-21	-27
		r	0	-10	-26	-40	-50
		γ_{rel}	0	-2	-6	-14	-23
		γ_{var}	-1	-5	-13	-21	-28
	30	$CorSim$	0	-1	-4	-6	-8
		r	0	-5	-13	-21	-25
		γ_{rel}	0	0	0	-2	-8
		γ_{var}	0	-2	-4	-6	-9
	100	$CorSim$	0	1	0	-2	-2
		r	0	-2	-5	-8	-9
		γ_{rel}	0	0	0	0	-2
		γ_{var}	0	-1	-1	-1	-2
100	10	$CorSim$	0	-5	-13	-21	-25
		r	0	-10	-25	-40	-50
		γ_{rel}	0	1	1	-4	-12
		γ_{var}	0	-5	-12	-20	-25
	30	$CorSim$	0	-1	-3	-5	-6
		r	0	-5	-13	-20	-25
		γ_{rel}	0	0	0	0	-4
		γ_{var}	0	-1	-3	-5	-7
	100	$CorSim$	0	0	-1	-1	-1
		r	0	-2	-5	-7	-9
		γ_{rel}	0	0	0	0	-1
		γ_{var}	0	0	-1	-1	-1

forms poorly, manifesting a negative bias as N and k decrease, and as λ and ρ increase. A similar pattern is seen in the Fisher's r-to-z transformation as applied to the γ_{rel} estimates. Rosner and Willett's (1988) formulas for calculating confidence intervals for γ_{rel} corrected estimates performs moderately well, manifesting the same patterns as Fisher's r-to-z transformations, but with a less extreme magnitude.

3.3 Simulation Discussion

The present simulations were designed to test the estimation and inferential performance of a variety of methods for calculating correlation amidst estimable measurement error. The data replicate the well-known negative estimation bias of raw correlation scores and suggest that 95% confidence intervals resulting from applying Fisher's r-to-z transformation to raw correlation scores must be treated with caution as these intervals may in fact be smaller than they should be, leading to a liberal inferential bias. Amongst the traditional approaches to ameliorating the deficiencies of raw correlation scores, the γ_{rel} and γ_{var} methods both achieved less biased estimates of ρ than the raw correlation, the former with slightly more consistency than the latter. However, these improvements were achieved at the cost of occasionally producing estimates of ρ that lay outside the reasonable bounds of -1 to +1. Applying Fisher's r-to-z transformation to γ_{rel} corrected estimates tended to yield 95% confidence intervals that were again too small. Rosner and Willett's (1988) formulas for calculating confidence intervals for γ_{rel} cor-

Table 7: ρ capture bias (Observed ρ capture rate - 95%) at $\lambda = .1$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	$CorSim$	0	0	1	1	5
		r	0	-1	1	0	-5
		γ_{rel}	0	-1	0	-1	-9
		γ_{var}	-4	-5	-4	-3	3
	30	$CorSim$	0	0	1	0	4
		r	0	0	0	0	0
		γ_{rel}	0	0	0	0	-6
		γ_{var}	-5	-5	-5	-4	3
	100	$CorSim$	0	1	1	1	3
		r	-1	0	0	0	0
		γ_{rel}	-1	0	0	0	-2
		γ_{var}	-5	-4	-5	-5	0
30	10	$CorSim$	1	0	1	1	3
		r	0	0	0	0	-34
		γ_{rel}	0	-1	0	-1	-19
		γ_{var}	-1	-2	-1	-1	4
	30	$CorSim$	0	0	0	0	2
		r	0	0	0	0	-5
		γ_{rel}	-1	0	0	-1	-6
		γ_{var}	-2	-2	-1	-2	3
	100	$CorSim$	0	0	0	0	0
		r	0	-1	0	0	0
		γ_{rel}	0	-1	0	0	-1
		γ_{var}	-1	-2	-2	-1	0
100	10	$CorSim$	0	1	1	1	-4
		r	-1	0	1	0	-87
		γ_{rel}	-1	0	0	-1	-19
		γ_{var}	-1	0	0	0	4
	30	$CorSim$	1	0	0	1	5
		r	0	0	0	0	-22
		γ_{rel}	0	0	0	0	-5
		γ_{var}	0	-1	0	0	2
	100	$CorSim$	1	0	0	0	5
		r	0	0	0	0	-2
		γ_{rel}	0	0	0	0	-1
		γ_{var}	0	-1	0	0	1

Table 8: ρ capture bias (Observed ρ capture rate - 95%) at $\lambda = 1$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	$CorSim$	0	0	1	2	5
		r	0	0	0	0	-80
		γ_{rel}	-4	-4	-5	-9	-48
		γ_{var}	-1	-1	1	2	-10
	30	$CorSim$	1	0	0	1	4
		r	0	0	-1	0	-36
		γ_{rel}	-1	-1	-3	-5	-20
		γ_{var}	-3	-3	-4	1	0
	100	$CorSim$	0	0	0	1	5
		r	0	0	0	1	-6
		γ_{rel}	-1	0	-1	-1	-10
		γ_{var}	-4	-4	-4	-3	3
30	10	$CorSim$	0	1	0	2	5
		r	-1	0	0	-9	-95
		γ_{rel}	-3	-3	-4	-11	-61
		γ_{var}	-1	0	0	2	-5
	30	$CorSim$	-1	1	0	1	5
		r	-1	0	0	-1	-90
		γ_{rel}	-2	-1	-1	-3	-39
		γ_{var}	-3	-1	-1	1	2
	100	$CorSim$	1	0	1	1	4
		r	0	0	0	0	-35
		γ_{rel}	0	0	0	0	-19
		γ_{var}	-1	-1	-1	0	4
100	10	$CorSim$	1	0	1	2	2
		r	0	-1	-4	-35	-95
		γ_{rel}	-2	-3	-4	-9	-71
		γ_{var}	0	-1	0	2	-7
	30	$CorSim$	1	0	0	1	2
		r	0	0	0	-5	-95
		γ_{rel}	-1	-1	-1	-3	-49
		γ_{var}	0	-1	0	1	3
	100	$CorSim$	0	0	0	1	-5
		r	-1	0	0	0	-87
		γ_{rel}	-1	0	0	0	-20
		γ_{var}	-1	0	0	0	4

Table 9: ρ capture bias (Observed ρ capture rate - 95%) at $\lambda = 10$

N	k	Method	$\rho = 0$	$\rho = .2$	$\rho = .5$	$\rho = .8$	$\rho = .99$
10	10	$CorSim$	0	0	1	0	-5
		r	0	-1	-6	-35	-95
		γ_{rel}	-18	-19	-19	-31	-88
		γ_{var}	5	5	4	-9	-53
	30	$CorSim$	1	1	1	2	3
		r	0	0	-1	-10	-94
		γ_{rel}	-12	-12	-12	-15	-75
		γ_{var}	5	4	2	-1	-29
	100	$CorSim$	0	0	1	2	5
		r	0	0	0	-1	-79
		γ_{rel}	-4	-4	-7	-9	-47
		γ_{var}	-1	0	1	1	-9
30	10	$CorSim$	1	0	-2	-7	-17
		r	1	-3	-28	-88	-95
		γ_{rel}	-30	-31	-35	-46	-91
		γ_{var}	4	3	-4	-18	-68
	30	$CorSim$	0	1	1	1	1
		r	0	-1	-7	-50	-95
		γ_{rel}	-12	-12	-16	-28	-83
		γ_{var}	0	1	1	-2	-35
	100	$CorSim$	0	0	0	2	5
		r	0	0	-1	-9	-95
		γ_{rel}	-3	-3	-4	-11	-66
		γ_{var}	-1	-1	-1	2	-7
100	10	$CorSim$	1	0	-9	-35	-66
		r	0	-12	-76	-95	-95
		γ_{rel}	-32	-31	-40	-58	-92
		γ_{var}	1	-1	-10	-45	-91
	30	$CorSim$	1	1	0	-3	-14
		r	0	-3	-27	-93	-95
		γ_{rel}	-10	-10	-14	-33	-85
		γ_{var}	0	0	0	-5	-51
	100	$CorSim$	1	1	1	1	2
		r	1	0	-3	-34	-95
		γ_{rel}	-2	-2	-3	-10	-69
		γ_{var}	0	0	1	2	-6

rected estimates, also generated overly liberal inferential tests, but to a lesser degree than the former two methods. The $CorSim$ method developed above performed similarly to the γ_{var} method in generating point estimates for ρ, but performed much better than all other methods at generating relatively unbiased 95% confidence intervals. This superior inferential performance, combined with the guarantee of point estimates falling within the bounds of -1 to +1, suggests that the $CorSim$ method may be favored by researchers investigating relationships amidst estimable measurement error.

Of peripheral note are the relative performance benefits from increasing N and k. A traditional perspective on data collection is that increasing N is a gold-standard means of improving statistical accuracy. However, the present results consistently argue against this position. Rejection rate of estimates generated by the γ_{rel} and γ_{var} methods appeared more sensitive to changes in k than in equivalent changes in N. Estimation bias of the raw correlation appeared sensitive to manipulation of k but rather insensitive to manipulation of N. Estimation bias of all correction methods were sensitive to both factors, but k more strongly so. While increasing k decreased ρ capture bias for all methods, increasing N appeared to increase ρ capture bias. The current results therefore suggest that experimentalists may achieve greater efficiency by focusing data collection resources on the repeated measurement of individuals rather than recruitment of numerous individuals.

4 General Discussion

The *CorSim* method has been presented above as an effective means of calculating correlation estimates and corresponding inferential intervals in the context of estimable measurement error. It should be noted also that the *CorSim* method is conveniently flexible. The simulations above test the performance of the *CorSim* method in the context of a simple two-variable correlational design, however there are often more complex cases in which one may seek to estimate and test a correlation, such as correlations between difference scores or correlations between linear slopes fit across a number of variables. The γ_{var} method has not yet been extended to facilitate application to such complex data sets, and while all of the remaining methods (*CorSim*, γ_{rel}, and raw correlation) may be applied in these circumstances, simulations in these contexts that are nearly identical in structure to that presented here maintain the above patterns of *CorSim* performance superiority (data available by request). From this one may conclude that the *CorSim* method has great promise to become the tool of choice within the domain of correlational investigations.

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